NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

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[Continued from June-July Number.]

XLVI. Fig. 27.
$ABLN$ is equivalent to $ABMK$ is equivalent to $ACIK$.
$NLFH = ABPO$ is equivalent to $BEDC$.
$\therefore$ $ABFH$ is equivalent to $ACIK + BEDC$.

XLVII. Fig. 27.
$ABLN$ is equivalent to $ACIK$.
$NLPO$ is equivalent to $STER$ is equivalent to $MTERC + QFD$.
$OPIH$ is equivalent to $REFH$ is equivalent to $REFQ + MBT$.
$\therefore$ $ABFH$ is equivalent to $ACIK + BEDC$.

XLVIII. Fig. 27.
$AVUH$ is equivalent to $2ACH$ is equivalent to $ACIK$.
$VBFU$ is equivalent to $2CBF$ is equivalent to $BEDC$.
$\therefore$ $ABFH$ is equivalent to $ACIK + BEDC$.
Wipper.

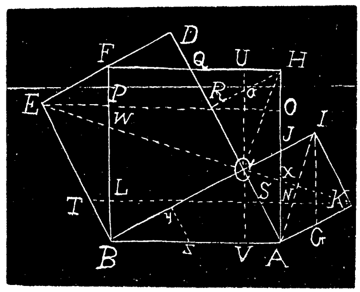

Fig. 27.

XLIX. Fig. 27.
$ABWX$, the half of $ABFH$, is equivalent to $ABC + CBW + CXA$.
But $ABC = BEF$ (is equivalent to $BWE + AXK$).
$\therefore$ $ABWX$ is equivalent to $CBE + CAK$.
$\therefore$ $ABFH$ is equivalent to $ACIK + BEDC$.

L. Fig. 27.
$Byz = FDQ$. $AzyC = AJIK$. $ARH = BEF$. $HRQ = ACJ$.
$\therefore$ $ABFH$ is equivalent to $ACIK + BEDC$.

LI. Fig. 27.
$ABC = BEF$. $CRa = FDQ$. $HRQ = IKG$. $HJCa$ is equivalent to $IGAJ$.
$\therefore$ $ABFH$ is equivalent to $ACIK + BEDC$.

That $HJCa$ is equivalent to $IGAJ$ is evident for the following reasons: $\triangle ACH$ is equivalent to $\triangle ACI$, having the same base, and equal altitudes.

Hence, subtracting $\triangle ACJ$, which is common to both, we have $\triangle CJH$ is equivalent to $\triangle AJI$.

$\therefore$ $HJCa$ is equivalent to $IGAJ$.

LII. Fig. 28.

$ABC=BEF$. $HRQ=ACJ$. $ARH=HKA$ is equivalent to $AKIJ+FDQ$.

$\therefore$ $ABFH$ is equivalent to $ACIK+BEDC$.

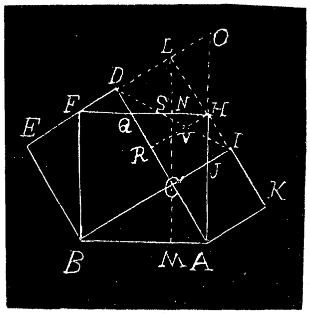

Fig. 28.

LIII. Fig. 28.

$AMNH$ is equivalent to $ACLH$ is equivalent to $ACIK$.

So, $MBFN$ is equivalent to $BEDC$.

$\therefore$ $ABFH$ is equivalent to $ACIK+BEDC$.

Wipper.

LIV. Fig. 28.

$CLOJ$ is equivalent to $CLHA$ is equivalent to $ACIK$.

$BFLC$ is equivalent to $BEDC$.

But $ABFH$ is equivalent to $BFOJ$.

$\therefore$ $ABFH$ is equivalent to $ACIK+BEDC$.

Hoffmann, 1800.

LV. Fig. 28.

$ABFH+BEF+FLH+HKA$ is equivalent to $ACIK+BEDC+ABC+CIL+CLD$.

$\therefore$ $ABFH$ is equivalent to $ACIK+BEDC$.

LVI. Fig. 28.

$ABC=BEF$. $ICD=AKH$ is equivalent to $AKIJ+FDQ$.

$SVH=SQD$, and $VHT=IJT$.

$\therefore$ By properly combining and substituting, $ABFH$ is equivalent to $ACIK+BEDC$.

LVII. Fig. 28.

$RDLH=ACIK$. $ARH=BEF$. $ABC=HFL$.

$\therefore$ $ABFH$ is equivalent to $ACIK+BEDC$.

[To be Continued.]

EUCLIDEAN GEOMETRY WITHOUT DISPUTED AXIOMS.

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(*a*)

PROPOSITION I. *If two straight lines in the same plane be perpendicular to the same straight line they are parallel.*

Prove by Axiom 11, and I, 27.*

*These and the subsequent numbers refer to the Book and Proposition in Todhunter's Euclid.